Section 14.2

Continuity for Two Variables

At what points (x, y) in the plane are the functions in Exercises 31–34 continuous?

32. a. $f(x, y) = \dfrac{x + y}{x - y}$ **b.** $f(x, y) = \dfrac{y}{x^2 + 1}$

Solution:

32. (a) All (x, y) so that $x \neq y$ (b) All (x, y)

Continuity for Three Variables

At what points (x, y, z) in space are the functions in Exercises 35–40 continuous?

37. a. $h(x, y, z) = xy \sin \dfrac{1}{z}$ **b.** $h(x, y, z) = \dfrac{1}{x^2 + z^2 - 1}$

Solution:

37. (a) All (x, y, z) with $z \neq 0$ (b) All (x, y, z) with $x^2 + z^2 \neq 1$

In Exercises 67 and 68, define $f(0, 0)$ in a way that extends f to be continuous at the origin.

68. $f(x, y) = \dfrac{3x^2y}{x^2 + y^2}$

Solution:

68. $\lim\limits_{(x,y)\to(0,0)} \dfrac{3xy^2}{x^2+y^2} = \lim\limits_{r\to 0} \dfrac{(3r\cos\theta)(r^2\sin^2\theta)}{r^2} = \lim\limits_{r\to 0} 3r\cos\theta\sin^2\theta = 0 \Rightarrow$ define $f(0,0) = 0$

Using the Limit Definition

Each of Exercises 69–74 gives a function $f(x, y)$ and a positive number ϵ. In each exercise, show that there exists a $\delta > 0$ such that for all (x, y),

$$\sqrt{x^2 + y^2} < \delta \quad \Rightarrow \quad |f(x, y) - f(0, 0)| < \epsilon.$$

73. $f(x, y) = \dfrac{xy^2}{x^2 + y^2}$ and $f(0, 0) = 0, \quad \epsilon = 0.04$

Solution

73. Let $\delta = 0.04$. Since $y^2 \le x^2 + y^2 \Rightarrow \frac{y^2}{x^2+y^2} \le 1 \Rightarrow \frac{|x|y^2}{x^2+y^2} \le |x| = \sqrt{x^2} \le \sqrt{x^2+y^2} < \delta \Rightarrow |f(x,y) - f(0,0)|$

$= \left|\frac{xy^2}{x^2+y^2} - 0\right| < 0.04 = \epsilon.$

Each of Exercises 75–78 gives a function $f(x, y, z)$ and a positive number ϵ. In each exercise, show that there exists a $\delta > 0$ such that for all (x, y, z),

$$\sqrt{x^2 + y^2 + z^2} < \delta \quad \Rightarrow \quad |f(x, y, z) - f(0, 0, 0)| < \epsilon.$$

78. $f(x, y, z) = \tan^2 x + \tan^2 y + \tan^2 z, \quad \epsilon = 0.03$

Solution:

78. Let $\delta=\tan^{-1}(0.1)$. Then $|x|<\delta, |y|<\delta$, and $|z|<\delta \Rightarrow |f(x,y,z)-f(0,0,0)|=\left|\tan^2 x+\tan^2 y+\tan^2 z\right|$

$\leq \left|\tan^2 x\right|+\left|\tan^2 y\right|+\left|\tan^2 z\right|=\tan^2 x+\tan^2 y+\tan^2 z<\tan^2\delta+\tan^2\delta+\tan^2\delta=0.01+0.01+0.01=0.03=\epsilon.$